3D Simulation and Inversion for Reconstructing Ground Surface Temperature Histories in Complex Settings

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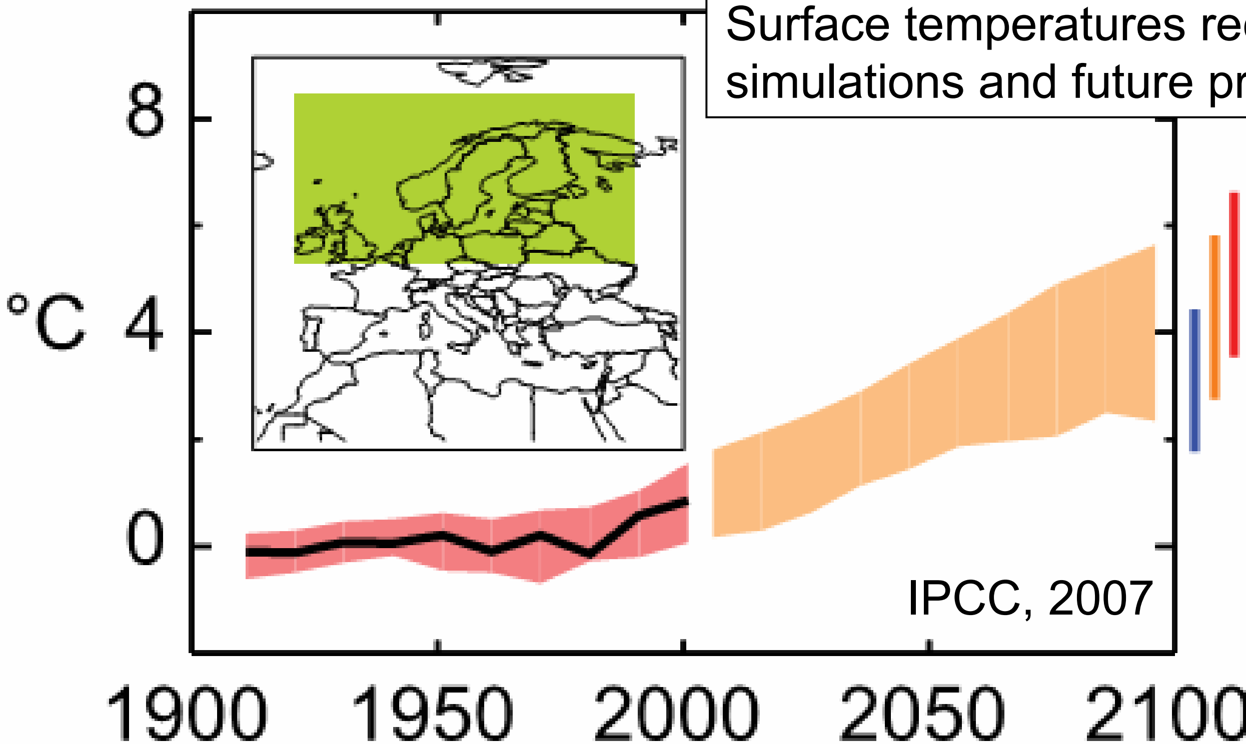
Climate & climate model predictions

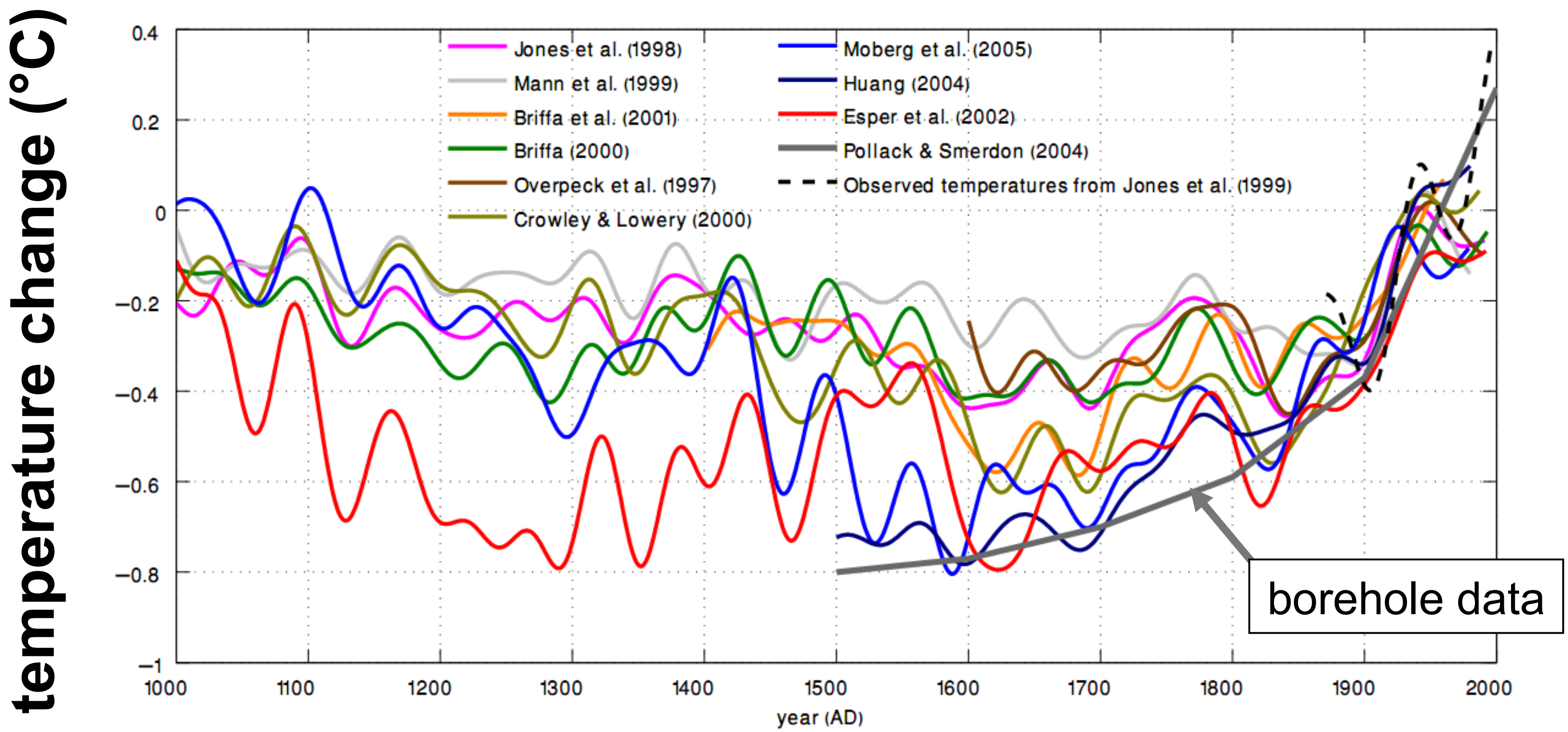
Surface temperatures records, simulations and future predictions



Uncertainty ranges for future predictions ~3.5°C

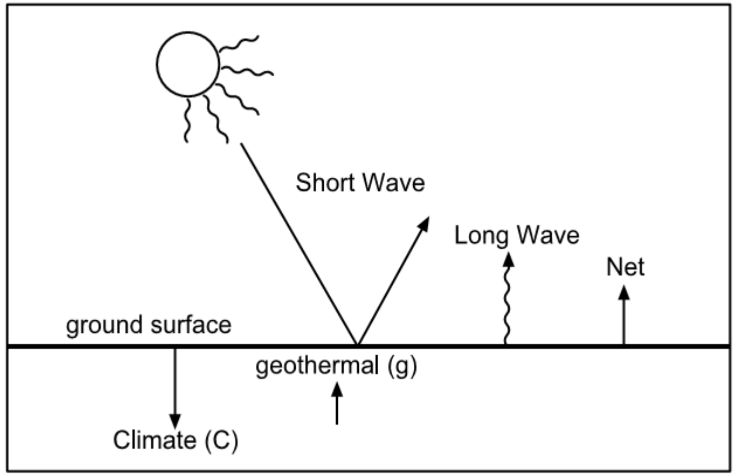
Temperature changes last 1000 years

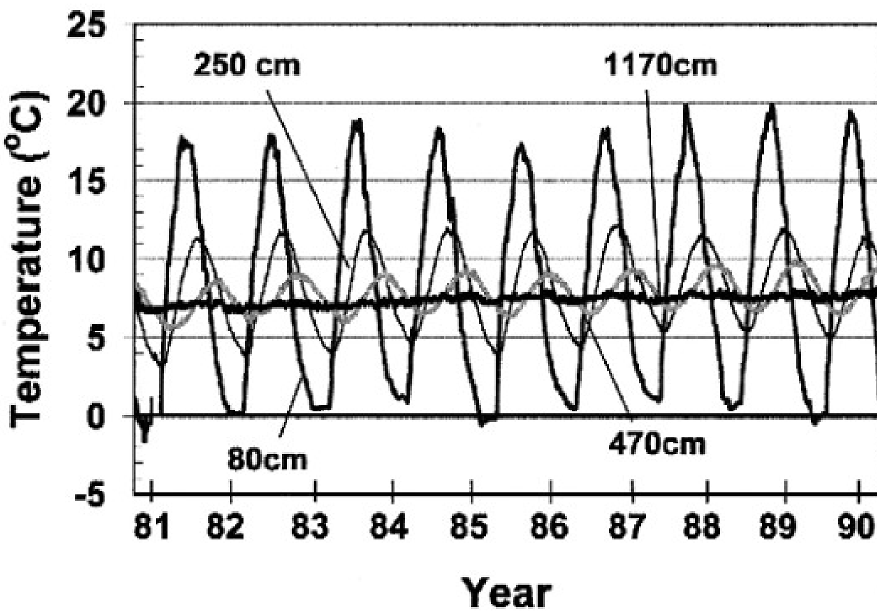
Colours = proxy data (tree rings, corals etc)



1000 AD - → **PRESENT DAY**

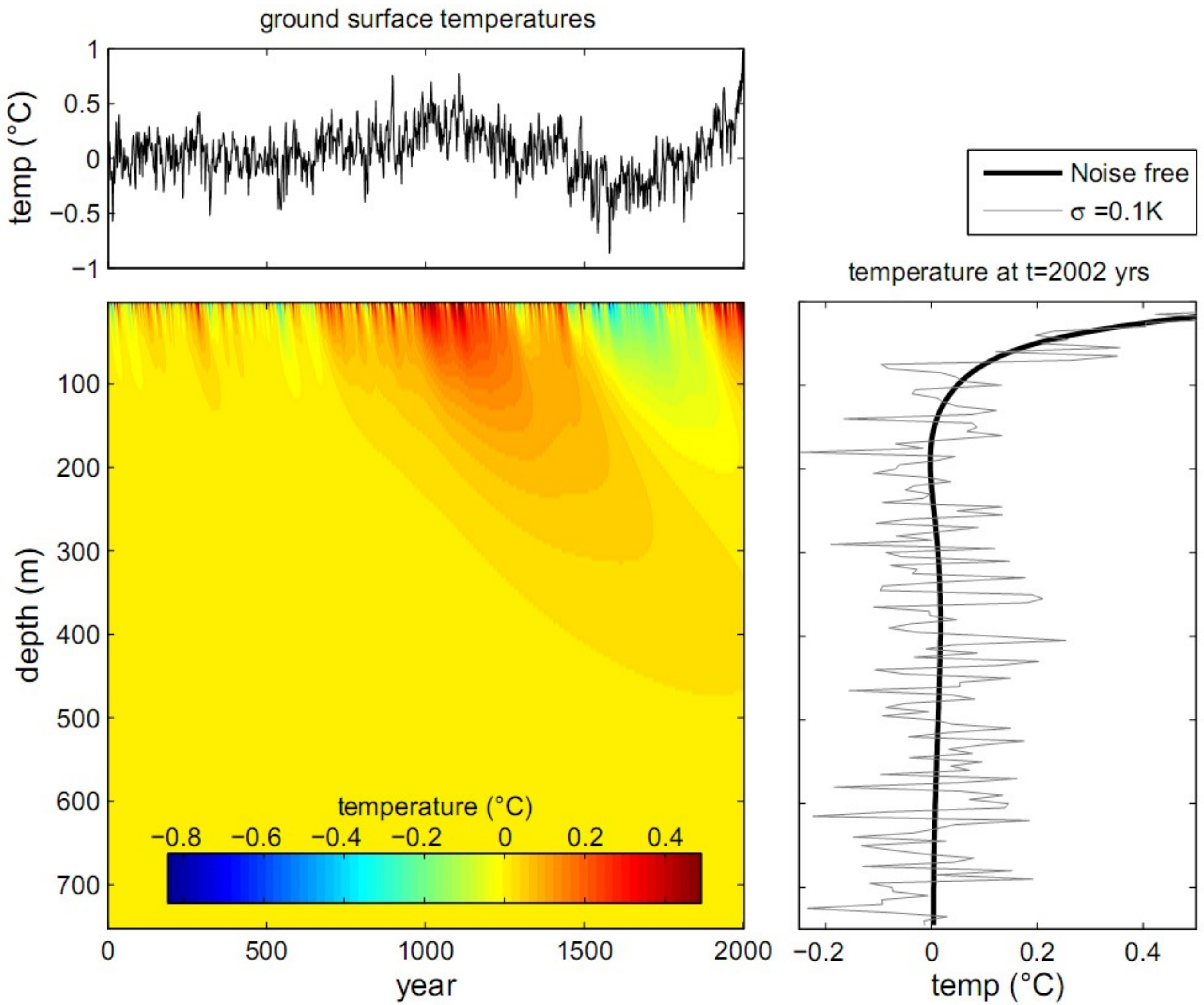
Ground surface energy balance



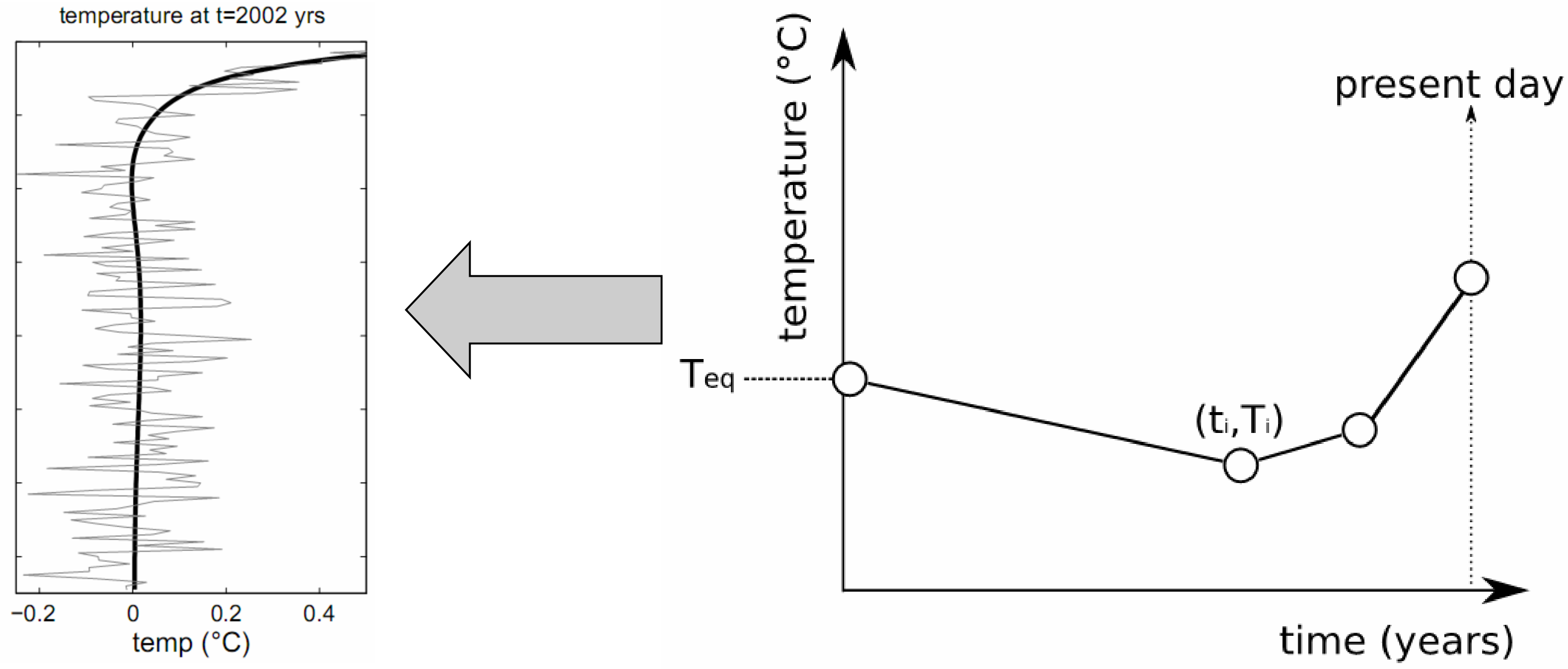


Deeper soil temperatures

Underground temperature perturbations



Model setup



We wish to relate a model of past temperature changes to the observed data.

This requires us to solve an **inverse problem** whereby the parameters of the model are estimated from the measured data.

Bayesian inference

- Bayes' Law: $\text{posterior} = \dfrac{\text{prior x likelihood}}{\text{evidence}}$

  Prior term encompasses model formulation and helps to constrain model parameters to physically reasonable values.

  Likelihood term is a measure of the model fit to the data.

  Evidence term used to compare different models and favours simpler models which adequately fit the data.

  Posterior term is then a fusion of the prior with all the information provided by the data.

Reversible Jump Markov chain Monte Carlo

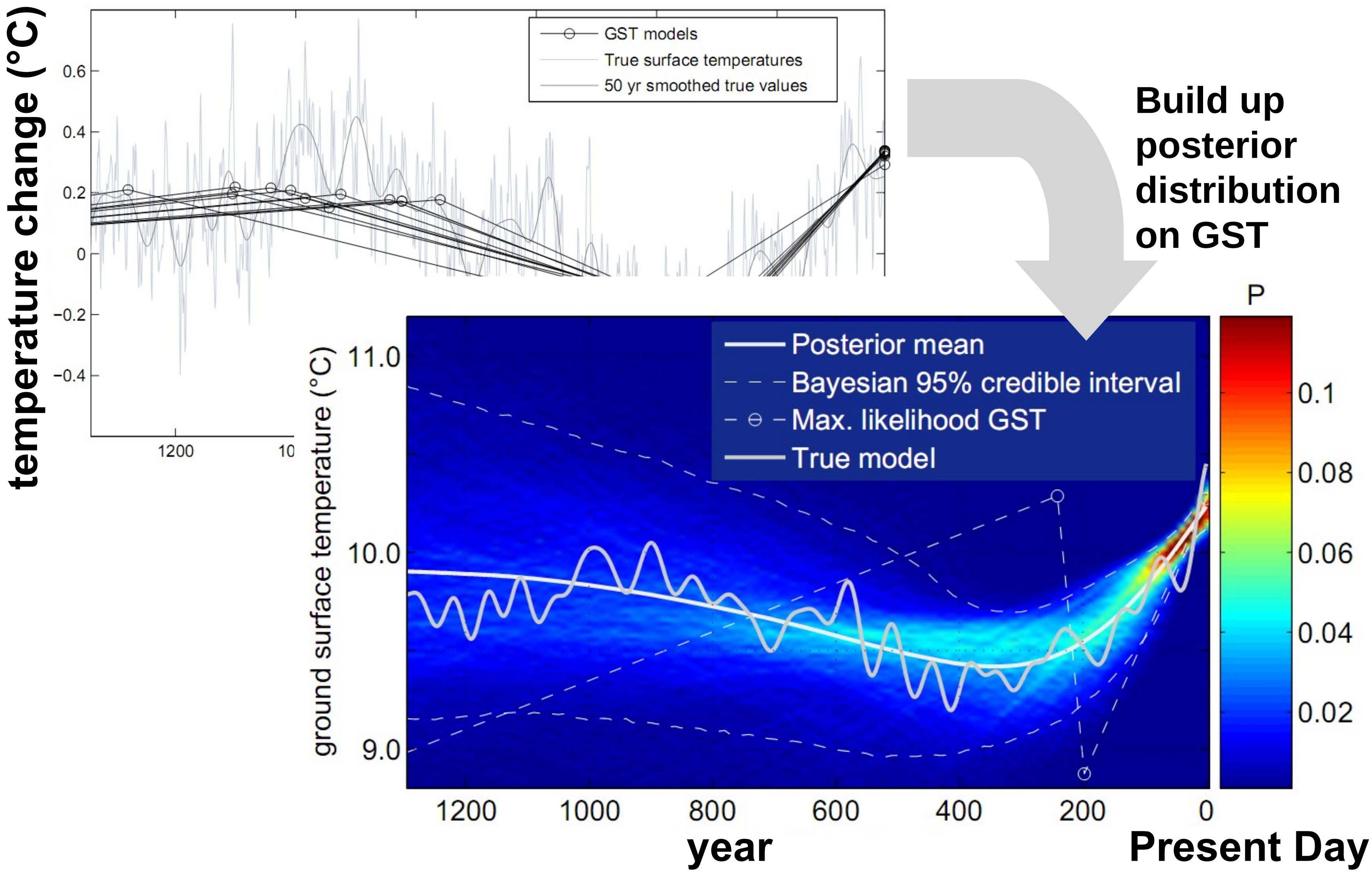
$\mathbf{m}_0$

Iterate

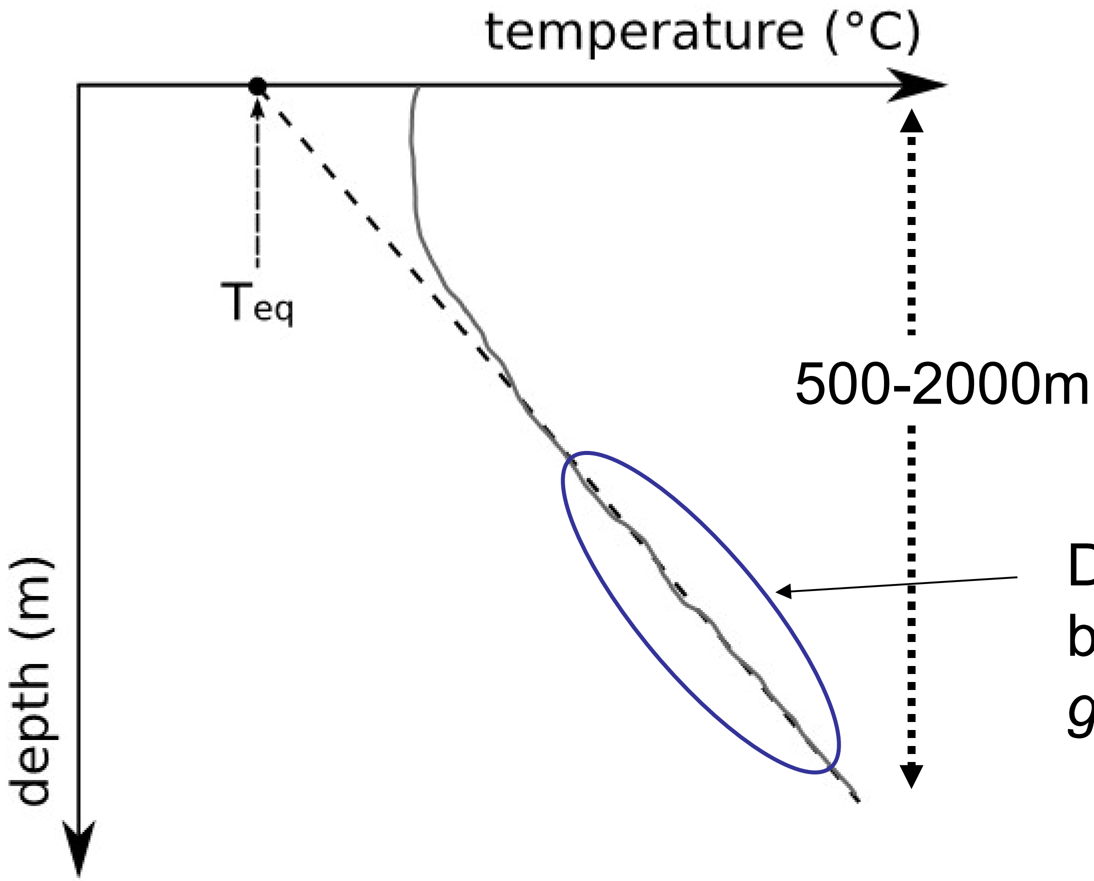
- Propose new **m**
- Calculate likelihood with new **m'**
- Accept new **m'** or retain current **m**

Sample iteratively from the posterior distribution of the model parameters conditioned on the data and the prior.

RJ-MCMC sampling: Synthetic test 1D

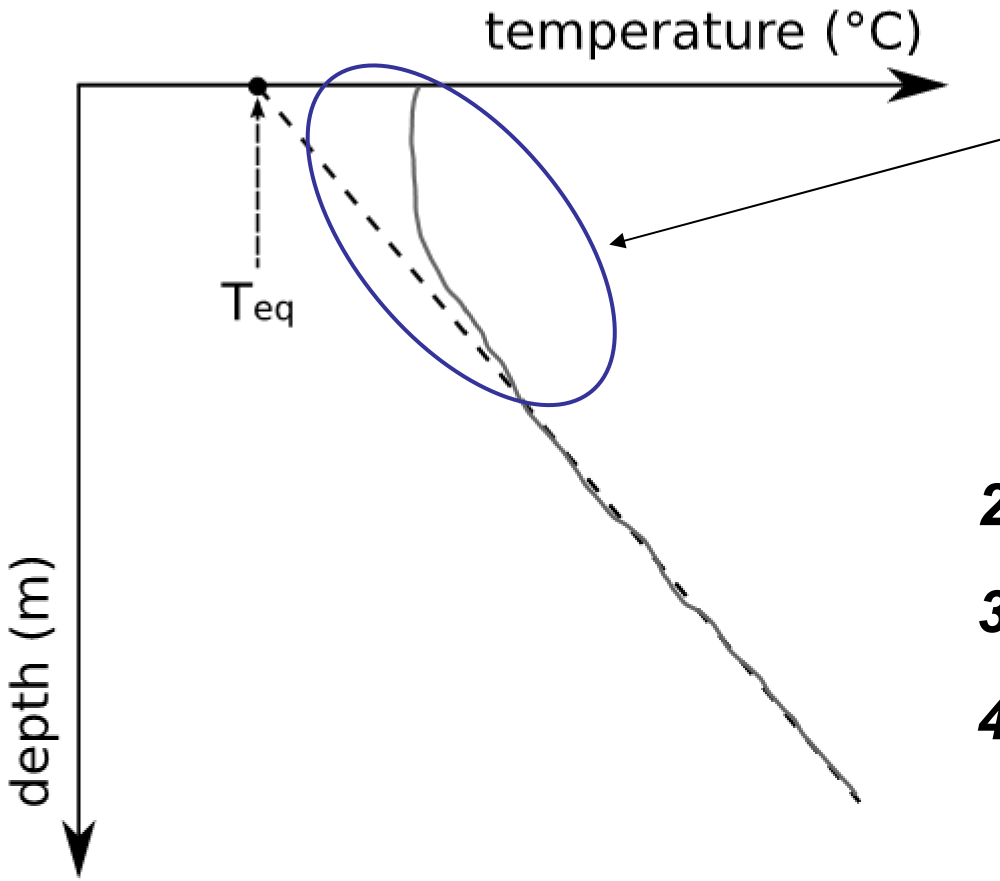


Data



Data acquisition has been focussed on *geothermal heat flux*

Data continued ….



Data at shallow depths are most significant for recent (500 year) climate reconstructions, but at these depths the effects of

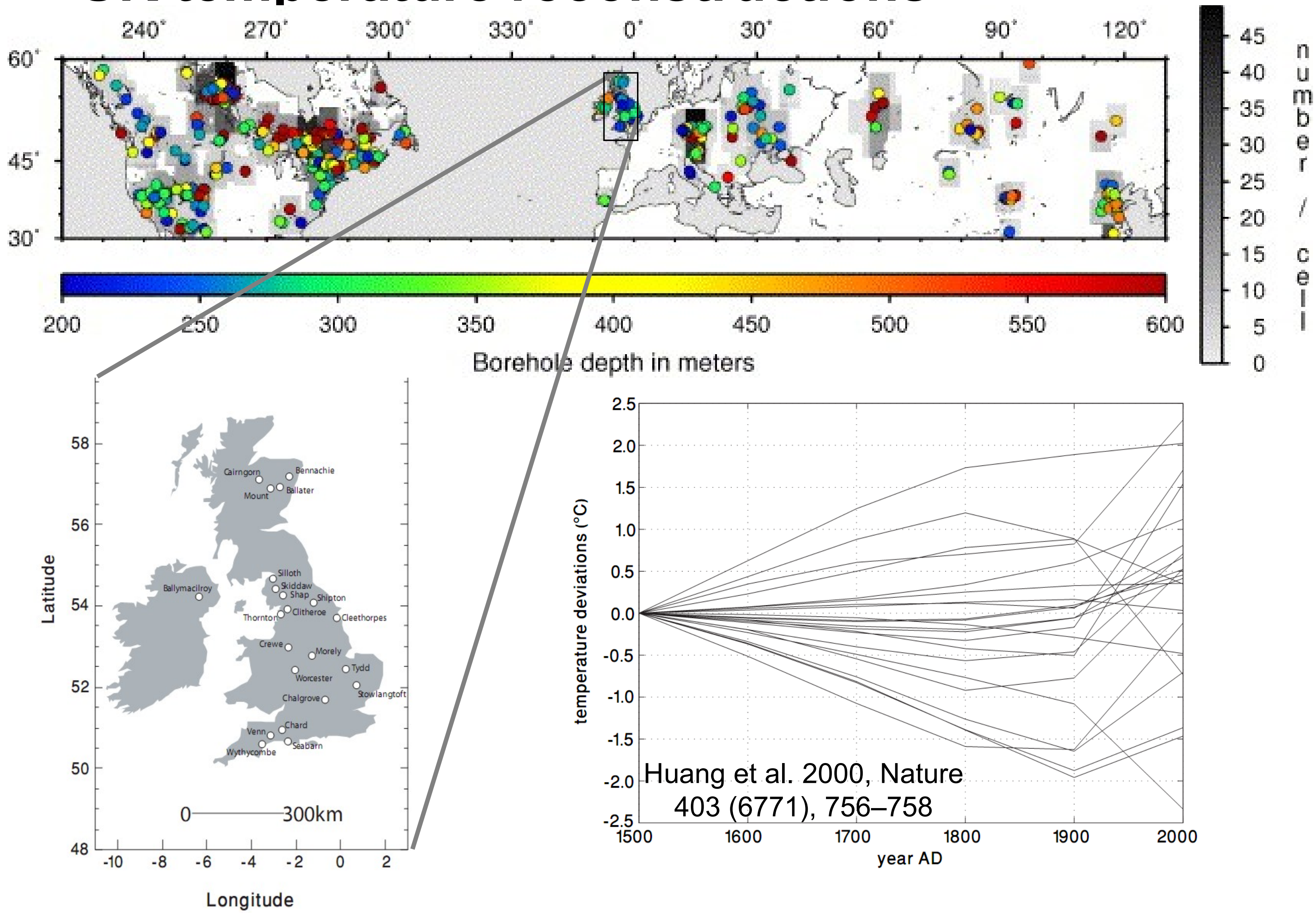
2) ***Topography***

3) ***Fluid flow***

4) ***Land use change***

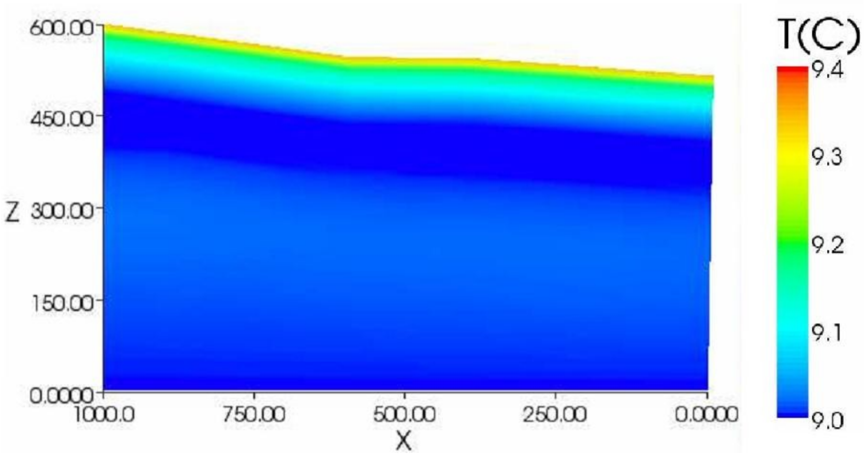
can also be significant.

UK temperature reconstructions

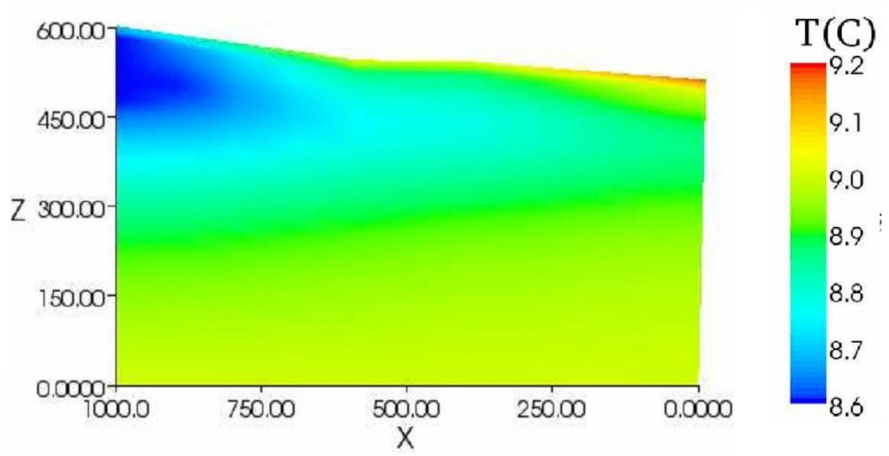


3D Finite element modelling

(a)



(b)



3D finite element solutions for the underground temperature perturbation caused by a 2000 year surface air temperature history

(b) No variation of ground temperature with altitude,

(c) 0.006K/km lapse rate.

Model reduction using POD

Snapshot data **S** is collected from output of the original FE simulation. The mean is subtracted to give:

$$\Psi_{ij} = \mathbf{S}_{ij} - \bar{\mathbf{S}}_j$$

We wish to find a finite set of basis functions to represent this data. To achieve this we calculate the singular value decomposition:

$$\Psi_{ij} = \mathbf{U}\Delta\mathbf{V}^T$$

Then the optimal representation can be found by:

$$\Phi = \Psi_{ij}\mathbf{U}_i/\sqrt{\lambda_i}, \quad i = 1, \ldots, k$$

The accuracy can be controlled by calculating the 'energy':

$$I(k) = \frac{\sum_{i=1}^{k} \lambda_i}{\sum_{i=1}^{n} \lambda_i}$$

The original FE model:

$$\mathbf{A}\mathbf{T}^{m+1} + \mathbf{B}\mathbf{T}^{m} = (1-\theta)\mathbf{f}^{m} + \theta\mathbf{f}^{m+1}$$

Now becomes:

$$\hat{\mathbf{A}}\mathbf{a}^{m+1} + \hat{\mathbf{B}}\mathbf{a}^{m} = (1-\theta)\hat{\mathbf{f}}^{m} + \theta\hat{\mathbf{f}}^{m+1}$$

$$\boxed{\begin{aligned}\hat{\mathbf{A}} &= \Phi^T\mathbf{A}\Phi \\ \hat{\mathbf{B}} &= \Phi^T\mathbf{B}\Phi \\ \hat{\mathbf{f}}^{m} &= \Phi^T\mathbf{f}^{m}\end{aligned}}$$

This new model is of dimension $k \times k$ where k is the number of retained singular values,

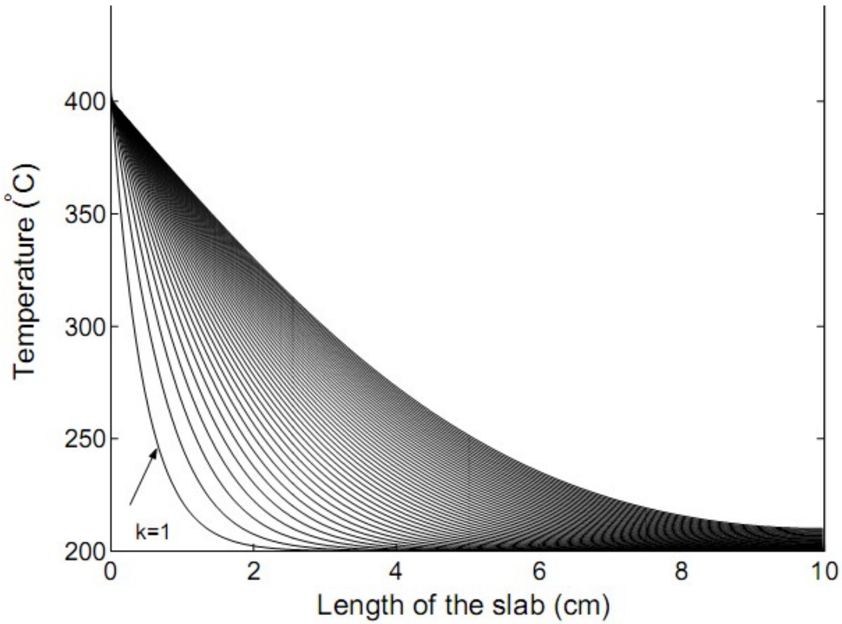
and crucially k<< the number of nodes in FE model.

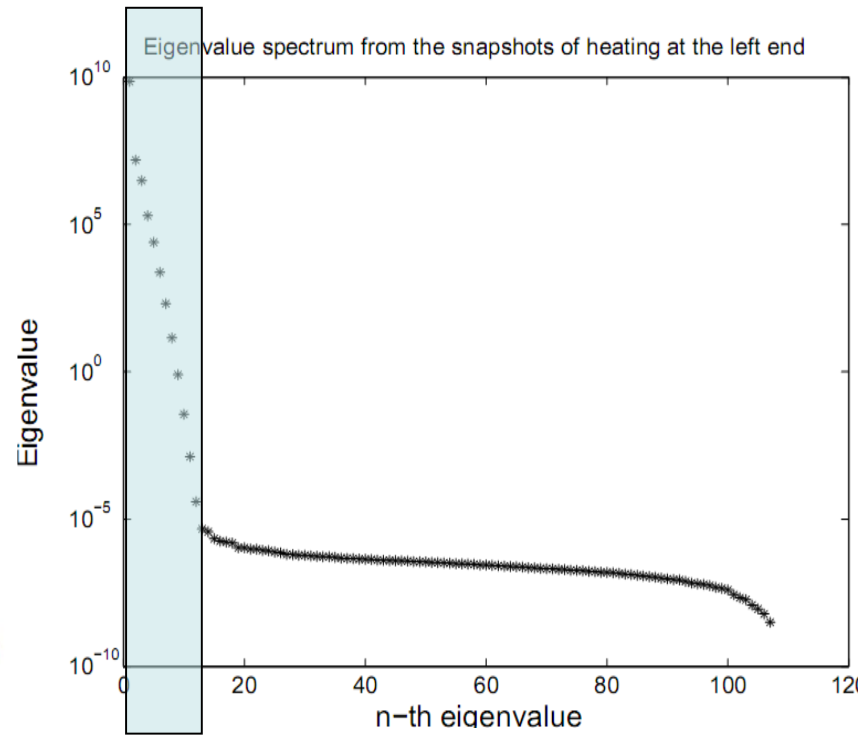
The full scale solution at the last time-step can then be calculated from the POD coefficients using:

$$\boxed{\mathbf{T} = \Phi\mathbf{a}}$$

Snapshot data & Eigenvalues

Heat conduction into a slab over time:



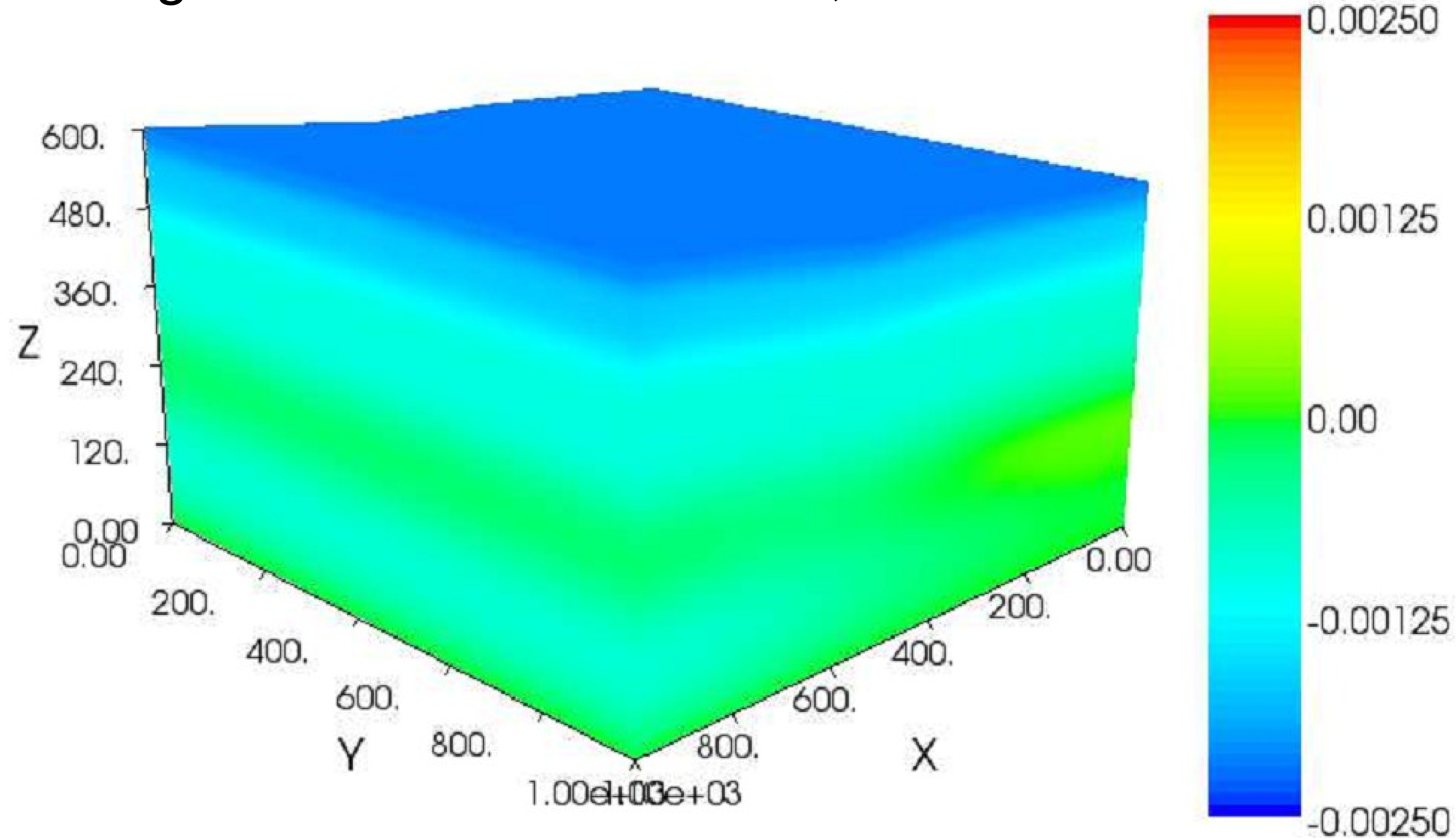


There is a sharp drop-off in the eigenvalues.

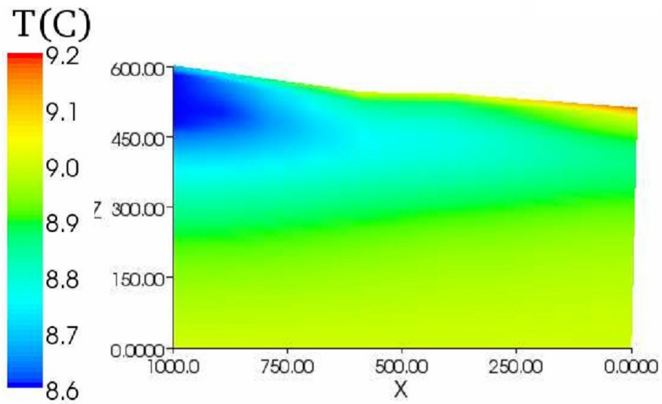
POD error

Difference between a POD and FE simulation: with 300x6yr time steps.

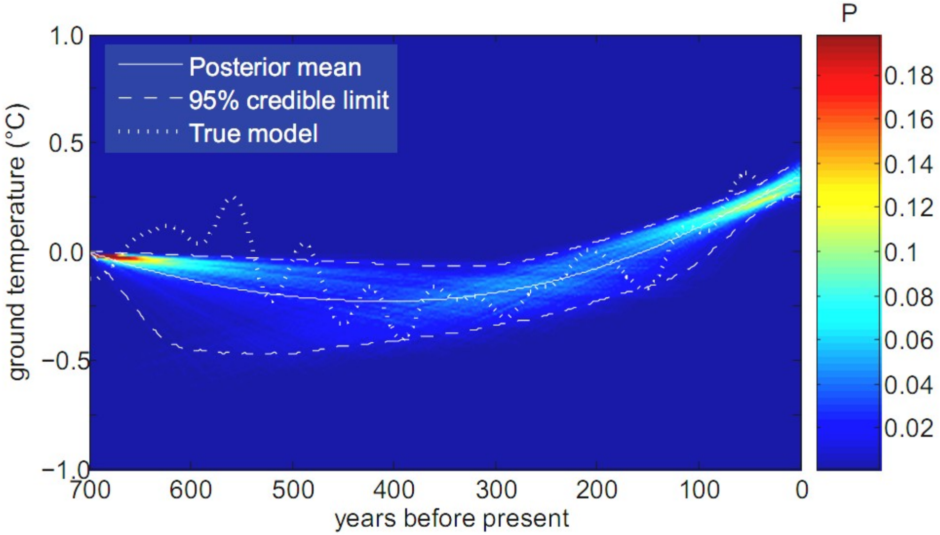
POD: 8 eigenvectors FE: 18,949 nodes



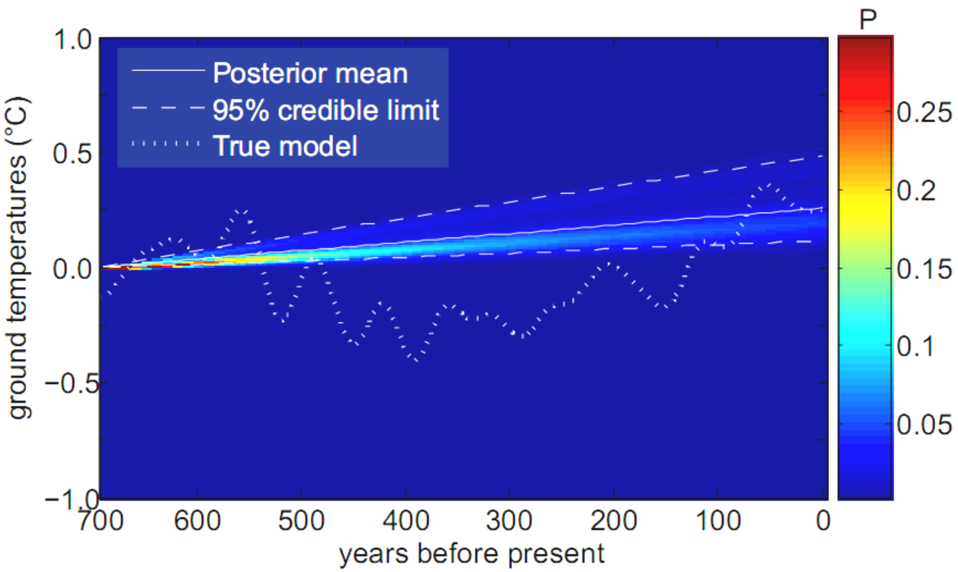
Comparing 1D/3D inversions



3D

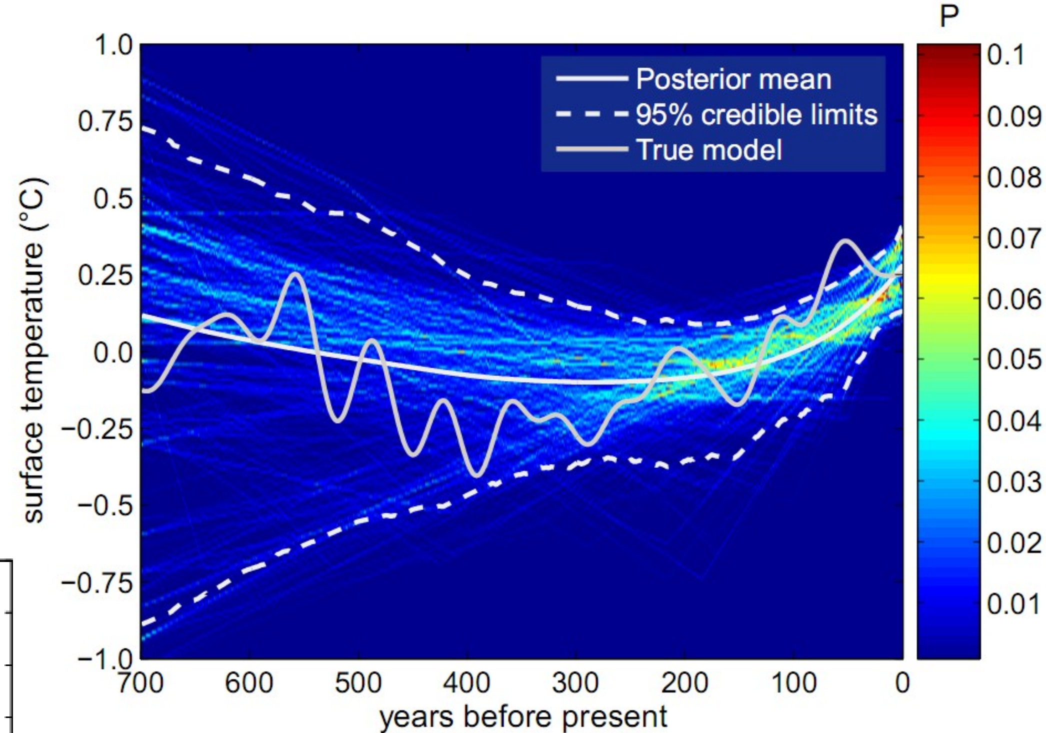


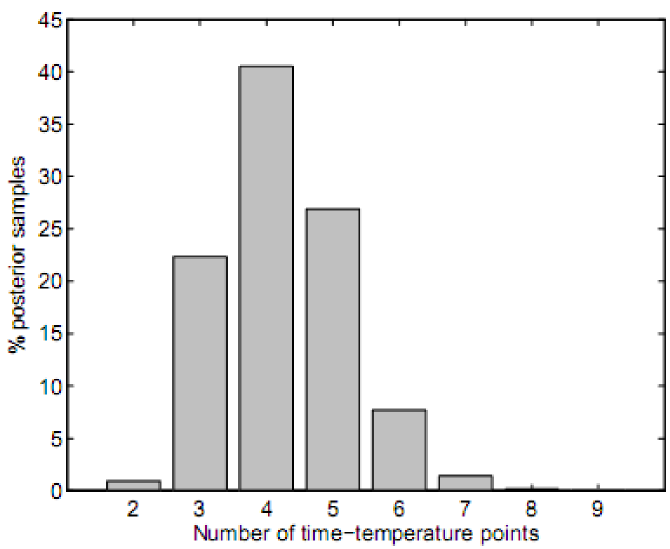
1D



Realistic synthetic example

Synthetic data with realistic noise also accounting for uncertain thermal conductivity in the volume.





Conclusions & Future work

- POD allows inversion of computationally expensive forward models
- Attention must be paid to error analysis (especially for non-linear simulations) so that the reduced order model solutions are valid.
- We would like to use Ordnance Survey Digital Elevation data for real data cases taking account of the measured topography.
- Future work could be to include underground fluid flow in the forward model.

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